AAPT | UNITED STATES PHYSICS TEAM

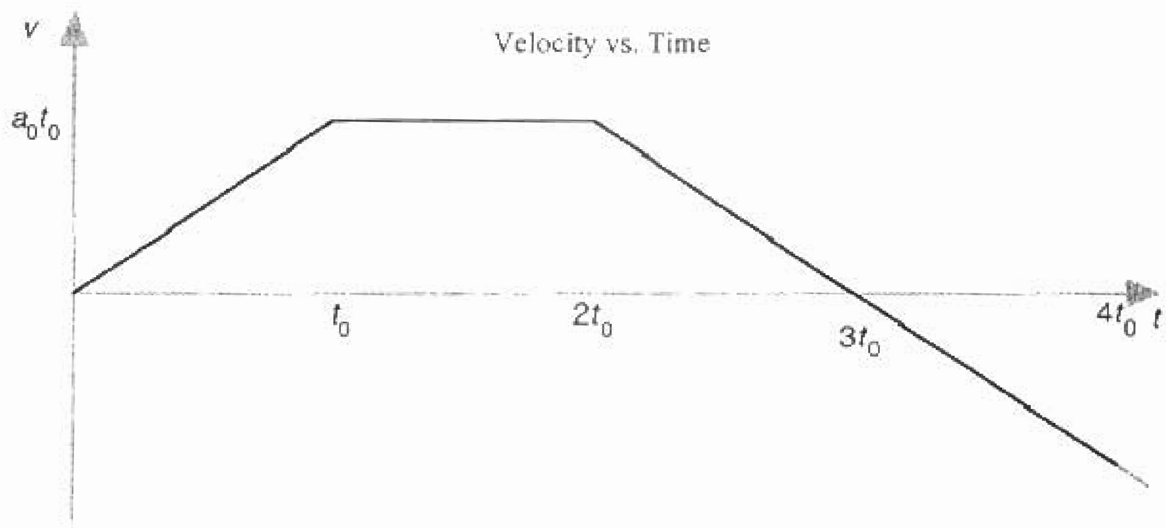
AIP | 2004

**Solutions to Problems**

*Any correct solution should be awarded equivalent points. Suggested partial-credit points are presented in square brackets at the right margin. You may further break down the listed points into one point increments. If it is clear they have done an intermediate step, they should get credit for it even if they have not presented it. For example, in 4.a., if a student wrote down $\omega = (Mv\alpha L)/(2mL^2)$, they should get 6 points credit. Students should not be penalized in a subsequent part for using the wrong answer to a previous part. (No double jeopardy.)*

<u>Points</u>

1. a. The graph should look like the following.




| | |
|---|---|
| Both axes are labeled | [1] |
| Shows $v = 0$ at $t = 0$ | [1] |
| Straight line with positive slope from $t = 0$ to $t = t_0$ | [1] |
| Shows discontinuity in slope at $t = t_0$ | [1] |
| Labels maximum velocity at $v = a_0t_0$ | [1] |
| Shows constant velocity from $t = t_0$ to $t = 2t_0$ | [1] |
| Shows discontinuity in slope at $t = 2t_0$ | [1] |
| Straight line with negative slope after $t = 2t_0$ | [1] |
| Shows $v = 0$ at $t = 3t_0$ | [1] |
| No spurious features shown | [1] |

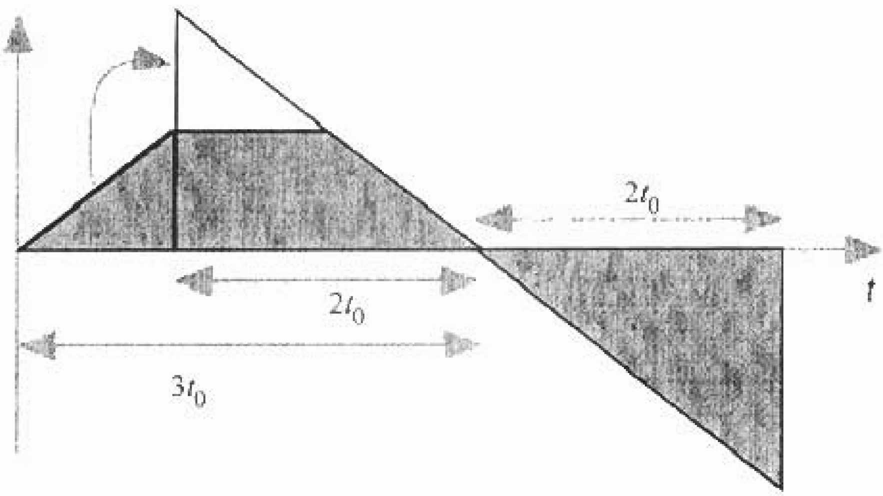
*{There are at least two methods that can be used to find the time at which the object returns to the origin. These are detailed below. Students may be awarded points from only one method.}*

b. Method I: The distance traveled is given by the area under the velocity vs. time curve. [5]
Continue the curve until the total area is zero, i.e., the area below the $t$-axis is equal to the area above it. Find the value of $t$ at which the object returns to the origin either by construction or geometrical formula. The construction method follows.

The area above the time axis is $d_+ = 2a_0t_0^2$. [4]

To make the area below the axis equal, extend the graph $2t_0$ beyond the zero crossing at $3t_0$. [4]

and $t = 2t_0 + 3t_0 = 5t_0$ [2]




*{Students should be given full credit for any valid geometrical argument.}*

Method II: For the interval $t = 0$ to $t = t_0$, $a = +a_0$ and $v_0 = 0$. The kinematic equations for motion with constant acceleration yield

At time $t = t_0$, $v = a_0t_0$ [2]

$$x = \tfrac{1}{2}a_0t_0^2.$$ [2]

These become the initial velocity and displacement for the next interval $t = t_0$ to $t = 2t_0$. During this interval, $a = 0$ and $v_0 = a_0t_0$.

At time $t = 2t_0$, $v = a_0t_0$ [2]

$$x = x_0 + v_0(2t_0 - t_0) = \tfrac{1}{2}a_0t_0^2 + (a_0t_0)t_0 = \tfrac{3}{2}a_0t_0^2.$$ [2]

These become the initial velocity and displacement for the next interval $t > 2t_0$. During this interval, $a = -a_0$ and $v_0 = a_0t_0$.

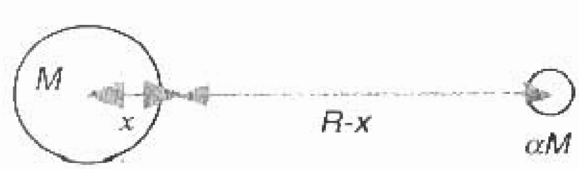
For $t > 2t_0$, $x = x_0 + v_0(t - 2t_0) - \tfrac{1}{2}a_0(t-2t_0)^2 = \tfrac{3}{2}a_0t_0^2 + a_0t_0(t-2t_0) - \tfrac{1}{2}a_0(t-2t_0)^2$ [4]

$$x = \tfrac{1}{2}a_0[3t_0 - (t-2t_0)][t_0 + (t-2t_0)] = \tfrac{1}{2}a_0(5t_0 - t)(t - t_0)$$

It returns to the origin when $x = 0$. This happens at $t = 5t_0$. [3]

The other solution is eliminated because the equation is only valid in the time interval $t > 2t_0$.

2. a. To find the center of mass of the star planet system, divide the distance $R$ into $R - x$ and $x$ as shown in the diagram. Placing the



origin at the center of mass, the general center of mass equation becomes $M_{tot}x_{cm} = m_1x_1 + m_2x_2$
$0 = Mx - \alpha M(R - x)$.

Solving for $x$, the center of star to center of mass distance. $x = \dfrac{\alpha R}{1+\alpha}$. [5]

Using Newton's Second Law to describe the star's motion $\sum F = Ma$ [2]

The net force is the universal gravitational force $F = G\dfrac{M\alpha M}{R^2}$ [3]

{*Maximum of* [2] *if* $x$ *is used in place of* $R$.}

For uniform motion in a circle of radius $x$ about the center of mass. $a = \dfrac{v^2}{x}$. [3]

{*Maximum of* [2] *if* $R$ *is used in place of* $x$.}

Substituting for $x$, this becomes $a = \dfrac{(1+\alpha)v^2}{\alpha R}$ [1]

Combining these equations $G\dfrac{M\alpha M}{R^2} = M\left(\dfrac{(1+\alpha)v^2}{\alpha R}\right)$ [2]

Canceling common factors $G\dfrac{\alpha M}{R} = \dfrac{(1+\alpha)v^2}{\alpha}$. [1]

Solving for $v$ $v = \sqrt{\dfrac{G\alpha^2 M}{R(1+\alpha)}}$. [1]

To lowest order in $\alpha$ this is $v \approx \alpha\sqrt{\dfrac{GM}{R}}$ [2] {*Exact answer required.*}

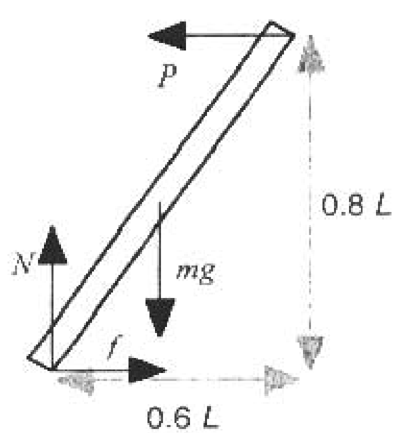
b. Solving for $R$ $R \approx \dfrac{\alpha^2 GM}{v^2}$ [2]

Inserting the values given with the problem

$$R \approx \frac{(3\times10^{-6})^2(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(2.0\times10^{30}\ \mathrm{kg})}{(3.0\ \mathrm{m/s})^2} = 1.33\times10^{8}\ \mathrm{m} \quad [2]$$

Converting to A.U. $R \approx 8.9\times10^{-4}$ A.U. [1]

3. a. Method I: (Inertial reference frame) The accompanying diagram shows the forces acting when the truck is accelerating to the right. The force of static friction must be to the right to provide the board's acceleration.




There is no acceleration in the $y$-direction, so the sum of forces in that direction must equal zero.

$$\sum F_y = N - mg = 0. \quad [2]$$

In the $x$-direction, Newton's Second Law yields

$$\sum F_x = f - P = ma. \quad [2]$$

There is no rotation about the center of mass (the center of the board), so the net torque about that point must equal zero. *{This is not true for all points. The condition that the net torque must be zero about any axis applies only when the object is in static equilibrium, i.e., not accelerating.}*

$$\sum \tau = P(0.4L) + f(0.4L) - N(0.3L) = 0.$$ [3]

Solving for $P$ $\quad P = 0.75N - f$ .

Substituting into the $F_x$ equation

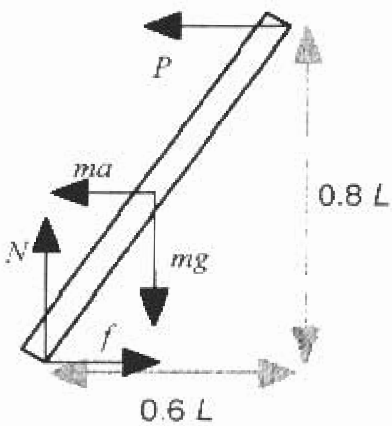
$$f - (0.75N - f) = 2f - 0.75N = ma.$$

The acceleration is maximum when $f$ takes on its maximum value $f_{max} = \mu_s N$ [2]

$$ma_{max} = 2f_{max} - 0.75N = 2\mu_s N - 0.75N = (2\mu_s - 0.75)mg$$

or $$a_{max} = (2(0.5) - 0.75)g = 0.25g$$ [1]

a. Method II: (Accelerating reference frame attached to truck) The accompanying diagram shows the forces acting when the truck is accelerating to the right. The force of static friction must be to the right to balance the other forces. There is no acceleration in the $y$-direction, so the sum of forces in that direction must equal zero.




$$\sum F_y = N - mg = 0.$$ [2]

In the $x$-direction, Newton's First Law yields

$$\sum F_x = f - P - ma = 0.$$ [2]

There is no rotation. In this frame any axis works. Selecting the bottom of the board.

$$\sum \tau = P(0.8L) + ma(0.4L) - mg(0.3L) = 0$$ [3]

Solving for $P$ $\quad P = 0.375mg - 0.50ma$ .

Substituting into the $F_x$ equation

$$f - (0.375mg - 0.50ma) - ma = f - 0.375mg - 0.50ma = 0.$$

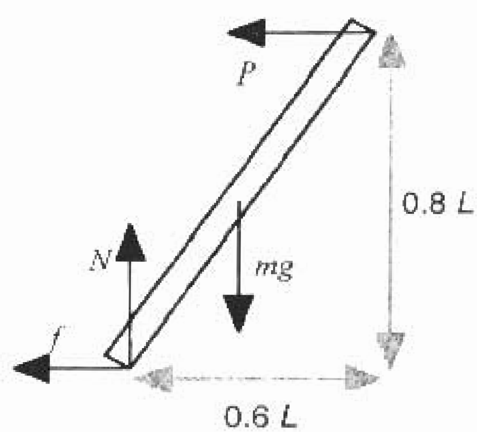
The acceleration is maximum when $f$ takes on its maximum value $f_{max} = \mu_s N = \mu_s mg$ [2]

$$ma_{max} = 2f_{max} - 0.75mg = 2\mu_s mg - 0.75mg = (2\mu_s - 0.75)mg$$

or $$a_{max} = (2(0.5) - 0.75)g = 0.25g$$ [1]

*{While we only present this method for Part (a), it can be used for Parts (b) and (c) as well.}*

b. For maximum acceleration in the stopping case, $f$ must be in the same direction as the acceleration. The equations become




$$\sum F_y = N - mg = 0$$ [2]

$$\sum F_x = f + P = ma$$ [2]

$$\sum \tau = P(0.4L) - f(0.4L) - N(0.3L) = 0$$ [3]

Solving for $P$ $\quad P = 0.75N + f$

Substituting into the $F_x$ equation

$$f + (0.75N + f) = 2f + 0.75N = ma.$$

The acceleration is maximum when $f$ takes on its maximum value $\quad f_{max} = \mu_s N$ [2]

$$ma_{max} = 2f_{max} + 0.75N = 2\mu_s N + 0.75N = (2\mu_s + 0.75)mg$$

or $$a_{max} = (2(0.5) + 0.75)g = 1.75g$$ [1]

c. If $f = 0$ in the Part b case, $P = ma$ [2]

and $P = 0.75N = 0.75mg$ [2]

Combining $a = 0.75g$ [1]

4. Before the collision the dumbbell is at rest and mass $M$ moves with velocity $v$. After the collision $M$ is at rest and the dumbbell is both translating and rotating. Let $v_{cm}$ equal the translational velocity of the dumbbell center of mass and let $\omega$ equal the angular velocity of the dumbbell about its center of mass.

The moment of inertia of the dumbbell about its center of mass is $I = 2mL^2$ [2]

Angular momentum is conserved. The initial angular momentum of $M$ about the dumbbell's center of mass equals the final angular momentum of the dumbbell about its center of mass.

$$Mv(\alpha L) = I\omega$$ [3]

Solving for ω $\omega = \frac{Mv\alpha L}{I} = \frac{Mv\alpha L}{2mL^2} = \frac{Mv\alpha}{2mL}$ [1]

Linear momentum is conserved. $Mv = 2mv_{cm}$ [3]

Solving for the center of mass velocity $v_{cm} = \frac{Mv}{2m}$. [1]

The collision is elastic, so kinetic energy is conserved.

$$\tfrac{1}{2}Mv^2 = \tfrac{1}{2}(2m)v_{cm}^{\ 2} + \tfrac{1}{2}I\omega^2$$ [3]

Substituting into this equation $Mv^2 = (2m)\left(\frac{Mv}{2m}\right)^2 + \left(2mL^2\right)\left(\frac{Mv\alpha}{2mL}\right)^2$ [1]

Simplifying $1 = \frac{M}{2m} + \frac{M\alpha^2}{2m} = \frac{M}{2m}\left(1+\alpha^2\right)$

Therefore $M = \frac{2m}{1+\alpha^2}$ [1]

b. If $\alpha = 1$ $M = \frac{2m}{1+1} = m$ [5]

{Immediately after the collision the lower mass $m$ has speed $v$. $M$ and the other $m$ are at rest. This conserves mechanical energy, linear momentum, and angular momentum.}

c. If $\alpha = 0$ $M = \frac{2m}{1+0} = 2m$ [5]

{Immediately after the collision both masses $m$ have speed $v$. This conserves mechanical energy and linear momentum. Angular momentum about the center of mass is zero in this case.}